## Unit 6 Ratios and Rates

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## Lesson 1

Introduction to Ratios and Unit Rate

## Vocabulary

## Ratio: A comparison of two quantities by division.

## Three ways to write a ratio: <br> 5 to 7 <br> $5: 7$ <br> $\frac{5}{7}$

Unit Rate: $\qquad$
Unit Price: $\qquad$

## PART I: Understanding Ratios

Nicholas asked members of his class if they go on Facebook on the weekends. The table below shows their answers. Use the table to answer the following questions.

| Sometimes on Facebook | Always on Facebook | Never on Facebook |
| :---: | :---: | :---: |
| 15 | 8 | 7 |

1. What is the ratio of the number of students who always go on Facebook to the number who never go on Facebook? $\qquad$
2. What is the ratio of the number of students who sometimes go on Facebook to the total number of students surveyed? $\qquad$
3. What does the ratio $30: 7$ represent?
4. What is the ratio of the number of female students to the number of male students in this class?
5. What is the ratio of the number of teachers to the number of students in this class?
6. What is the ratio of the number of female students to the number of the students in the class?

## Equivalent Ratios

7. The ratio of the number of wrestlers to the number of football players at Sachem is 16 to 48. Represent this ratio as a fraction: $\qquad$ . You can use what you know about fractions to find equal ratios and write the ratio in simplest form. Write an equivalent ratio: $\qquad$ .
8. Write three ratios that are equal to the given ratio 12: 21. (Express this ratio as a fraction to help you.)
$\qquad$ equivalent ratio 2 : $\qquad$ equivalent ratio 3 : $\qquad$

## Express each ratio as a fraction in simplest form:

9) 27 rooms to 48 windows
10) 3 gallons to 15 quarts
11) 7 baseballs to 14 softballs
12) 24 DVDs to 16 CDs

## PART II: Unit Rate

13) 216 meters in 8 seconds (meters per second)
14) $\$ 10$ for 8 fish (Money per fish)
15) $\$ 2,702$ for 28 people (Money per person)
16) 60 feet in 15 seconds (Feet per second)

## Which is the better price?

17) Six candy bars for $\$ 2.62$ or eight bars for $\$ 3.40$ ?
18) A dozen paint brushes for $\$ 6.46$ or eighteen for $\$ 9.90$ ?

## Try These:

1) Mr. DeMeo needs help solving this problem. A hot dog truck sells 9 hot dogs for $\$ 11.25$.
a) Find the unit price.
b) If he wants to buy 3 hot dogs for Mrs. DiJorio, how much will it cost?
2) Write in simplest form: 13 diamonds to 52 cards
3) Which is the better price? 32 ounces for $\$ 3.84$ or 40 ounces for $\$ 4.40$

Classwork: Find the unit rate of each
4) The Seneca's Student Government sold $\$ 75$ worth of tickets for a talent show in 3 hours. How much money did they make in one hour?
5) At Six Flags, 1,473 people entered the park in 3 hours. How many people entered the park in 1 hour?
6) A wedding a Villa Lombardi's cost $\$ 9,750$ for 150 people. How much does Villa Lombardi's charge per guest?

## Lesson 1: Homework (2 pages)

Find the unit rate.

1) If a runner ran 102 meters in 12 seconds, how many meters did he/she run per second?
2) Ticketmaster sold 1200 tickets to the Mets-Yankees game in 3 hours. How many tickets were sold per hour?

Which is the better bargain? Find the unit price for each and compare them.
3) Pens: $\$ 4.50$ for 30 pens
or
$\$ 3.20$ for 20 pens
4) Pencils: 16 for $\$ 8.32$ or 35 for $\$ 17.15$
5) Lucy went away on vacation for 10 days and when she came home she had 280 emails. How many emails did she get per day?
6) Derek just got a new I-Phone and downloaded 348 songs in 6 hours. How many songs did he download per hour?
7) Ryan and his brother are comparing the prices of two brands of cereal. Frosted Flakes costs $\$ 2.25$ for a 15 ounce box. Lucky charms costs $\$ 3.90$ for a 30 -ounce box. Which brand is more expensive and by how much per-ounce?
8) Gas mileage is the average number of miles you can drive a car per gallon of gasoline. A test of a new car resulted in 2,250 miles being driven using 125 gallons of gas. Find the new car's gas mileage.
9) The table shows the prices that Mr. Alfredson paid at 3 different gas stations. Complete the table to determine which gas station had the better price per gallon.

| Gas Station | Gallons | Price | Price per Gallon (Show work here) |
| :---: | :---: | :---: | :---: |
| Hess | 15 | $\$ 43.50$ |  |
| Coastal | 10 | $\$ 29.40$ |  |
| Amoco | 12 | $\$ 35.88$ |  |

## Lesson 2

## Unit Rate Continued/ Ratios with Fractions

Do Now:

1) Write the following minutes as fractions of an hour in simplest form:
a) 30 minutes
b) 45 minutes
c) 15 minutes
d) 50 minutes
2) Simplify: $2 \frac{4}{7} \div 1 \frac{3}{6}$

## Vocabulary

- 

A fraction whose numerator or denominator is a fraction is called a complex fraction.

- A unit rate is a rate which is expressed as $A / B$ units of the first quantity per 1 unit of the second quantity for two quantities A and B.

For example: If a person walks $21 / 2$ miles in $1 \frac{1}{4}$ hours at a constant speed, then the unit rate is $21 / 2 \div 1 \frac{1}{4}=2$. The person walks 2 mph .

## Examples: Find the unit rate of the following

1) $8 \frac{1}{4}$ miles in $2 \frac{1}{5}$ hours. (Miles per hour)
2) Sarah used $3 \frac{1}{4}$ gallons of gas driving $58 \frac{1}{2}$ miles (Miles per gallon)
3) One lap around a dirt track is $\frac{1}{3}$ mile. It takes Cole $\frac{1}{9}$ hour to ride one lap. What is Cole's unit rate (miles per hour) around the track?
4) Mr. Oakes wants to make a shelf with boards that are $1 \frac{1}{3}$ feet long. If he has an 18 foot long board, how many pieces can he cut from the big board?

## Try These:

1) Which car can travel further on 1 gallon of gas?

Blue Car: Travels $18 \frac{2}{5}$ miles using 0.8 gallons of gas Red Car: Travels $17 \frac{2}{5}$ miles using 0.75 gallons of gas
2) Sally is making a painting for which she is mixing red paint and blue paint. The table below shows the different mixtures being used.

| Red Paint <br> (Quarts) | Blue Paint <br> (Quarts) | Unit Rate |
| :---: | :---: | :--- |
| $1 \frac{1}{2}$ | $2 \frac{1}{2}$ |  |
| $2 \frac{2}{5}$ | 4 |  |
| $3 \frac{3}{4}$ | $6 \frac{1}{4}$ |  |
| 4 | $6 \frac{2}{3}$ |  |
| 1.2 | 2 |  |
| 1.8 | 3 |  |

a) Fill in the Unit Rates to determine how much Red paint is used for each quart of Blue paint in the mixtures.
b) Is the amount of blue paint proportional to the amount of red paint? (Are all unit rates equal?)
3) During their last workout, Izzy ran $21 / 4$ miles in 15 minutes and her friend Julia ran $33 / 4$ miles in 25 minutes. Each girl thought she were the faster runner. Based on their last run, which girl is correct?
4) For Anthony's birthday his mother is making cupcakes for his 12 friends at his daycare. The recipe calls for $31 / 3$ cups of flour. This recipe makes $21 / 2$ dozen cupcakes. Anthony's mother has only 1 cup of flour.
A) How many cupcakes are $21 / 2$ dozen?
B) Is there enough flour for each of his friends to get a cupcake? Explain and show your work.
5) The local bakery made 3 batches of cookies this morning. If the bakery used 5.25 cups of flour, how much flour was needed for each batch?

1) You are getting ready for a family vacation. You decide to download as many movies as possible before leaving for the road trip. If each movie takes $1 \frac{2}{5}$ hours to download and you downloaded for $5 \frac{1}{4}$ hours, how many full movies did you download?
2) A toy remote control jeep is $121 / 2$ inches wide while an actual jeep is pictured to be $183 / 4$ feet wide. What is the value of the ratio of the width of the remote control jeep to width of the actual jeep?
3) Jason eats 10 ounces of candy in 5 days.
a. How many ounces will he eat per day?
b. How long will it take Jason to eat 1 pound of candy? $(16$ ounces $=1$ pound $)$
4) $\frac{1}{3}$ cup of flour is used to make 5 dinner rolls.
A) How much flour is needed to make one dinner roll?
B) How many cups of flour are needed to make 3 dozen dinner rolls?
C) How many rolls can you make with $5 \frac{2}{3}$ cups of flour?

## Lesson 3

## Equal Ratios and Proportions

## Vocabulary

1. Ratio: A comparison of two quantities by division. Can be written as $\frac{a}{b}, a: b$, or $a$ to $b$. $(\mathrm{b} \neq 0)$
2. Proportion: An equation that shows that two ratios are equivalent (equal).
3. Cross Product: In two ratios, the cross products are found by multiplying the denominator of one ratio by the numerator of the other. If you cross multiply and the products are the same then it is proportional.
*If two ratios form a proportion, if the cross products are equal.

## Examples:

Check if each pair of ratios form a proportion. Write $=$ if the ratios are proportional. If they are not, write $=$.

1) $\frac{9}{8} \bigcirc \frac{3}{2}$
2) $6: 33 \bigcirc 2: 11$
3) 8 to $45 \bigcirc 40$ to 9

Do these form a true proportion? Write yes or no.

| 4) $\frac{10}{15}=\frac{15}{10}$ | $5) 6: 12$ and $14: 28$ | $\frac{6)}{\frac{16 \text { Blue }}{17 \text { Red }}, \frac{32 \text { Blue }}{34 \text { Red }}}$ | 7 ) 3 to 8 and 15 to 32 |
| :--- | :--- | :--- | :--- |

Rule: To solve Proportions

1) $\qquad$
2) $\qquad$

## Examples:

The following ratios are proportional...solve for the variable.

1) $\frac{33}{12}=\frac{11}{n}$
2) $n$ to 2 and 9 to 1
3) $32: y$ and $8: 9$
4) $\frac{6 \frac{3}{10}}{x}=\frac{126}{42}$
5) $\frac{5}{11}=\frac{x+4}{22}$
6) $\frac{y}{9.3}=\frac{12.6}{5.4}$

## Try These:

The following ratios are proportional...solve for the variable.

1) $\frac{10}{n}=\frac{6}{45}$
2) $1.7: 2.5$ and $3.4: d$
3) $\frac{2}{2 x+1}=\frac{2}{9}$
4) $\frac{7}{3}=\frac{a}{3}$
5) $m$ to 28 and 5 to 14
6) $\frac{a+3}{5}=\frac{2}{2}$
7) 
8) $\frac{x+3}{5}=\frac{2}{3}$
9) 

$\frac{5 x}{3}=\frac{10}{12}$

$$
\frac{3 x+1}{8}=\frac{4}{2}
$$

11) $\frac{\frac{2}{5}}{x}=\frac{4}{2}$
12) $7: 3$ and $x: 210$
13) $\frac{x+2}{3}=\frac{2 x-4}{5}$
14) 5 to 3 and $x$ to 15
15) $45: 9$ and $30: x$
16) $\frac{x-3}{3}=\frac{2 x-7}{5}$

Do these form a true proportion? Write yes or no.

| 1) $\frac{20}{50}=\frac{80}{150}$ | 2) $7: 12$ and $21: 35$ | $\frac{3) \frac{9 \text { Blue }}{17 \text { Red }}, \frac{36 \text { Blue }}{68 \text { Red }}}{}$ | 4 ) 3 to 8 and 12 to 32 |
| :--- | :--- | :--- | :--- |

Solve the following proportions for the given variable.

| 5) $\frac{1}{3}=\frac{4}{x}$ | 6 ) 2 to 15 and $x$ to 45 | 7 ) $x: 8$ and $2: 4$ |
| :--- | :--- | :--- |
| 8) $\frac{40}{100}=\frac{x}{20}$ | 9) 18 to 12 and 6 and $x$ | 10) $\frac{x}{21}=\frac{9}{7}$ |
| 11$) \frac{3}{4}=\frac{x}{60}$ | 12) $\frac{x}{12}=\frac{11}{6}$ | 13) $24: x$ and $3: 5$ |
| 14$) \frac{x}{9}=\frac{12 \frac{3}{5}}{5}$ |  | $15 \frac{16.9}{5}$ |

## Steps:

1. Write ratio in word form
2. Set up proportion from word problem
3. Solve for x
4. Check to see if answer makes sense

## Examples:

1) There are 5 dogs for every 3 cats in the pet store. If there are 20 dogs at the store, how many cats are at the store?

$$
\frac{50 Y A R D S}{1 \mathrm{~min}}=\frac{x Y A R D S}{3 \mathrm{~min}}
$$

$\underline{\text { Dogs }}$
Cats
2) On a map 2 inches represent 40 miles. On the map, the distance from City A to City B is 9 inches. What is the actual distance?
3) A building is 36 meters tall and casts a shadow of 12 meters. At the same time, how long would the shadow of a tree which is 4 meters tall be?
4) The corresponding sides of similar triangles are proportional. Triangles $A B C$ and $A^{\prime} B^{\prime} C^{\prime}$ are similar. Find $x$ and $y$.
A



## Try These:

1) Nick takes 3 hours to read 240 pages. At this rate, how many pages can he read in 2 hours?
2) Nancy is 5 feet tall. At a certain time of day, she measures her shadow, and finds it is 9 feet long. She also measures the shadow of a building which is 200 feet long. How tall is the building?
$\qquad$
3) Ryan can run 4 blocks in 2 minutes. How long does he take to run 8 blocks at the same speed?
4) A ball dropped from a tall building falls 16 feet in the first second. How far does it fall in 2 seconds?
5) A burro is standing near a cactus. The burro is 60 inches tall. His shadow is 4 feet long. The shadow of the cactus is 7 feet long. How tall is the cactus?
6) Mr. Green can grow 12 tomato plants in a 3 square foot area. How many tomatoes can he grow if he has a 15 square foot area?
7) Making 5 apple pies requires 2 pounds of apples. How many pounds of apples are needed to make 15 pies?
8) Jack and Jill went up the hill to pick apples and pears. Jack picked 10 apples and 15 pears. Jill picked 20 apples and some pears. The ratio of apples and pears picked by both Jack and Jill were the same. Determine how many pears Jill picked.

## For each of the following - set up a proportion WITH LABELS and then solve.

1. It takes 18 people in Mrs. Haines's math class to pull a 35 ton bus. How many people would it take to pull a 140 ton bus?
$\frac{\text { people }}{\text { tons }}$
2. Physics tells us that weights of objects on the moon are proportional to their weights on Earth. Suppose a 180 lb man weighs 30 lbs on the moon. What will a 60 lb boy weigh on the moon?
3. Joe is at Stop and Shop and sees that 9 oranges cost $\$ 3.00$. He needs 27 oranges to make orange juice. How much money will 27 oranges cost?
4. A sample of 96 light bulbs consisted of 4 defective ones. Assume that today's batch of 6,000 light bulbs has the same proportion of defective bulbs as the sample. Determine the total number of defective bulbs made today.
5. Marissa is 5 feet tall. At a certain time of the day, she measures her shadow and finds that it is 9 feet long. She also measures the shadow of a building which is 225 feet long. How tall is the building?
6. On a map, one inch represents 30 miles. How many inches represent 480 miles?
7. The corresponding sides of similar triangles are proportional. Triangles ABS and $\mathrm{A}^{\prime} \mathrm{B}^{\prime} \mathrm{C}^{\prime}$ are similar. Find $x$.


## Lesson 5

## Proportions with Charts

Steps to find missing quantities in a ratio table where a total is given:

1) Determine the unit rate from the ratio of two given quantities
2) Use it to find the missing quantities in each equivalent ratio

## Examples:

1) For a school field trip there must be a proportional relationship between the number of adults and the number of students. Complete the table.

| Number of <br> Students | Number of <br> Adults | Total People |
| :---: | :---: | :---: |
|  | 2 |  |
| 36 | 3 |  |
|  |  | 52 |
| 120 |  |  |

2) Students in 6 classes, displayed below, ate the same ratio of cheese pizza slices to pepperoni pizza slices. Complete the following table, which represents the number of slices of pizza students in each class ate.

| Slices of <br> Cheese Pizza | Slices of <br> Pepperoni Pizza | Total Pizza |
| :---: | :---: | :---: |
| 6 | 15 | 7 |
| 8 | $13 \frac{3}{4}$ |  |
| $3 \frac{1}{3}$ |  |  |
|  |  | $2 \frac{1}{10}$ |

## Try These:

1) a) Complete the following table

| Distance <br> Ran <br> (miles) | Distance <br> Biked <br> (miles) | Total <br> Amount of <br> Exercise <br> (miles) |
| :---: | :---: | :---: |
| $3 \frac{1}{2}$ | 7 | 6 |
| $2 \frac{1}{8}$ | $3 \frac{1}{3}$ |  |
|  |  |  |

b) What is the relationship between distances biked and distances ran?
2) The following table shows the number of cups of milk and flour that are needed to make biscuits. Complete the table.

| Milk <br> (cups) | Flour (cups) | Total (cups) |
| :---: | :---: | :---: |
| 7.5 |  |  |
|  | 10.5 |  |
| 12.5 | 15 |  |
|  |  | 11 |

3) The table below shows the combination of dry prepackaged mix and water to make concrete. The mix says for every 1 gallon of water stir 60 pounds of dry mix. We know that 1 gallon of water is equal to 8 pounds. Using the information provided in the table, complete the remaining parts of the table.

| Dry Mix <br> (pounds) | Water <br> (pounds) | Total (pounds) |
| :---: | :---: | :---: |
|  | 8 |  |
| 75 | 10 |  |
|  |  | $14 \frac{1}{6}$ |
| $4 \frac{1}{2}$ |  |  |

1) Complete the following table. Show your work.

| Blue <br> Cups | Red <br> Cups | Total <br> Cups |
| :---: | :---: | :---: |
|  | 3 |  |
| 4 |  |  |
| 6 | 9 |  |
|  |  | 20 |
|  |  |  |

2) To make green paint, students mixed yellow paint with blue paint. The table below shows how many yellow and blue drops from a dropper several students used to make the same shade of green paint.

Complete the table.

| Yellow <br> $(\mathrm{Y})$ <br> $(\mathrm{ml})$ | Blue <br> $(\mathrm{B})$ <br> $(\mathrm{ml})$ | Total |
| :---: | :---: | :---: |
| $31 / 2$ | $5^{1 / 4}$ |  |
|  |  | 5 |
|  | $63 / 4$ |  |
| $61 / 2$ |  |  |

If ABC ~ DEF, write a proportion and solve for the missing side.
3.

5. Create a proportion from each set of numbers. Only use 4 numbers from each set of numbers.
a) $9,3,2,6$
b) $4,32,1,8$
c) $9,12,8,6$

Lesson 6

## Vocabulary:

Reduction: $\qquad$
Enlargement/Magnification: $\qquad$
Scale Drawing: reductions or enlargements of two dimensional drawings, not actual objects.
Scale: The constant ratio of each actual length to its corresponding length in the drawing. This scale can be expressed as the scale factor. ("new over original"..."Scale drawing over actual")

1) Can you determine if the following are enlargements or reductions?


## Reduction or Enlargement

B)

C)


Reduction or Enlargement
2) What are possible uses for enlarged drawings/pictures?
3) What are possible uses for reduced drawings/pictures?

Identify if the scale drawing is a reduction or enlargement of the actual picture.
4) a. Actual
b. Scale Drawing

5)
a. Actual

b. Scale Drawing

Reduction or Enlargement
Reduction or Enlargement
6)

b. Scale Drawing


Reduction or Enlargement

b. Scale Drawing


Reduction or Enlargement

## Drawing Geometric Figures

To draw geometric figures to scale, use your knowledge of similar figures and proportional relationships. The ratios of corresponding side lengths of similar figures are equal to the scale factor, so the scale factor indicates how many times larger or smaller to make each side length in the scale drawing.

## Determine if the following figure will get enlarged (magnified) or reduced if the scale factor is:

8) scale factor is 3
9) scale factor is $\frac{2}{3}$
10) scale factor is $5: 3$
11) scale factor is 2

## Examples:

Determine the scale factor for the following proportional shapes:
12)

10 m
Scale Drawing (new)

5 m
13) Original


14) Celeste drew an outline of a building for a diagram she was making and then drew a second one mimicking her original drawing. Fill in the table below and then answer the following questions.


|  | Height | Length | Area(HxL) |
| :--- | :--- | :--- | :--- |
| Original <br> Drawing |  |  |  |
| New <br> Drawing |  |  |  |

a) Is this a reduction or an enlargement?
b) What is the scale factor? $\qquad$
c) What is the area of the original rectangle?
$\qquad$ square units
d) What is the area of the new rectangle?
$\qquad$ square units
e) What is the ratio of the area of the scale rectangle to the area of the original rectangle? $\qquad$
f) How does the ratio of the area relate to the scale factor? $\qquad$
15)Luca drew and cut out small right triangles for a mosaic piece he was creating for art class. His mother really took a liking and asked if he could create a larger one for their living room and Luca made a second template



|  | Height | Length |
| :--- | :--- | :--- |
| Original <br> Drawing |  |  |
| New <br> Drawing |  |  |

Does a scale factor exist? If so, what is it? If not, explain.

To find the ratio of the areas in two figures you must SQUARE the scale factor!!
16) If the scale factor of the sides is 2 , then the ratio of the area is $\qquad$ -.
18) If the scale factor of the sides is $\frac{4}{3}$,
then the ratio of the area is $\qquad$ .
19) If the sides of a rectangle are doubled (multiplied by 2 ), how does the area change?
20)


Scale factor: $\qquad$

Actual Area: $\qquad$

Scale Drawing Area: $\qquad$

Ratio of Scale Drawing Area to Actual Area: $\qquad$
21) Scale factor: $\frac{2}{3}$ (Write as fraction.) bigger or smaller?

Draw the new triangle.

22) Scale factor: $\frac{5}{3}$ bigger or smaller?

Draw the new polygon.

23)


Scale factor: $\qquad$
Actual Area $=$ $\qquad$
Scale Drawing Area = $\qquad$
Ratio of Scale Drawing Area to Actual Area: $\qquad$

Describe the relationship between the scale factor and the ratio of areas? $\qquad$

## Use the figures below to answer questions 1 and 2.



1. What is the scale factor for the parallelograms?

2. What is the perimeter of the original drawing?
A. $\frac{9}{25}$
A. 15 ft
B. $\frac{4}{9}$
B. 50 ft
C. $\frac{3}{5}$
C. 25 ft
D. $\frac{5}{3}$
D. 30 ft

## Determine the scale factor:

3) Original
3 m

Scale Drawing (new)

3 m
4) Original

5) If the scale factor of the sides is 5 , then the ratio of the area is $\qquad$ .
6) If the scale factor of the sides is $\frac{1}{2}$, then the ratio of the area is $\qquad$ .
7) If the scale factor of the sides is $\frac{7}{9}$, then the ratio of the area is $\qquad$ .
8) If the sides of a rectangle are tripled (multiplied by 3 ), how does the area change? $\qquad$
9) 



|  | Height | Length |
| :--- | :--- | :--- |
| Original <br> Drawing |  |  |
| Scale <br> Drawing |  |  |

a) Is this a reduction or an enlargement?
b) What is the scale factor? $\qquad$
c) What is the area of the original rectangle? $\qquad$ square units
d) What is the area of the new rectangle? $\qquad$ square units
e) What is the ratio of the area of the scale rectangle to the area of the original rectangle? $\qquad$
f) How does the ratio of the area relate to the scale factor? $\qquad$
10) Make a scale drawing of the figure using the given scale factor.

Scale factor: $\frac{1}{2}$


|  | Height | Length |
| :--- | :--- | :--- |
| Original <br> Drawing |  |  |
| Scale <br> Drawing |  |  |

a) Is this a reduction or an enlargement?
b) What is the area of the original rectangle? $\qquad$ square units
c) What is the area of the new rectangle? $\qquad$ square units
d) What is the ratio of the area of the new rectangle to the area of the original rectangle? $\qquad$
e) How does the ratio of the area relate to the scale factor? $\qquad$

